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A VARIANT OF THE METHOD OF CORRECTION OF THE ELASTOPLASTIC PROPERTIES OF COMPOSITES USING ESTIMATES OF THE BINDING OF CONSTITUENTS

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Along with the nonlinearity of local constitutive equations, the limited statistical information on the structure of multicomponent composites is a handicap to the exact solution of the problem of determining the effective plastic properties of multicomponent composite materials. Therefore, in calculating the macroscopic characteristics of composites, one can ether determine their upper and lower bounds (the Khashin–Shtrikman fork) [1] or obtain approximate models. One method of allowing for data on the composite structure and increasing the accuracy of the calculation model is the expansion of moment functions in a series [2]. This, however, requires, a knowledge of the multipoint correlation moments, and this is very difficult to obtain in practice [3].

Another method of designing a refined model for the macroscopic behavior of a composite involves adequate estimation of the binding of the constituents, which depends on the well-founded choice of parameters of a reference body [4].

In this paper, we propose a variant of statistical averaging of the equilibrium equations for an elastoplastic, isotropic, multicomponent composite to construct its macroscopic constitutive equations. To describe the geometrical features of the composite structure, a parameter that characterizes the degree of binding of the constituents is introduced into the original constitutive relations. This allows one to describe, using the proposed method, the nonlinear hardening of the composite beyond the elastic limit and to calculate its effective characteristics.

The system of macroscopic constitutive equations for an elastoplastic isotropic composite formed by n different constituents is determined by statistical averaging of the equilibrium equation in displacements and has the form [4]

$$\langle s_{ij}\rangle = 2\mu^*(e_s)\langle e_{ij}\rangle, \qquad \langle \sigma_{pp}\rangle = 3K^*(e_s)\langle \varepsilon_{pp}\rangle, \tag{1}$$

where

$$\mu^* = \mu\frac{(1-\alpha)\xi}{1-\alpha\xi}; \qquad \xi = \sum_{s=1}^{n}\frac{c_s\mu_s(e_s)}{\mu(1-\alpha)+\alpha\mu_s(e_s)};$$

$$K^* = K\frac{(1-\gamma)\eta}{1-\gamma\eta}; \qquad \eta = \sum_{s=1}^{n}\frac{c_sK_s}{K(1-\gamma)+\gamma K_s}; \qquad e_s = \frac{(1-\alpha)\mu+\alpha\mu^*}{(1-\alpha)\mu+\alpha\mu_s(e_s)}e;$$

σ_{ij} and ε_{ij} are the stress- and strain-tensor components; $s_{ij} = \sigma_{ij} - (1/3)\delta_{ij}\sigma_{pp}$; $e_{ij} = \varepsilon_{ij} - (1/3)\delta_{ij}\varepsilon_{pp}$; $\mu_s(e_s)$ is the plasticity modulus of the sth constituent, K_s is its bulk modulus ($K_s = \text{const}$); c_s is the volume content of the sth constituent; μ and K are the reference-body parameters; $\alpha = 2(4-5\nu)/[15(1-\nu)]$; $\gamma = (1+\nu)/[3(1-\nu)]$; $\nu = (3K-2\nu)/(6K+2\mu)$; $e_s = \sqrt{\langle e_{kl}\rangle_s\langle e_{kl}\rangle_s}$; and $e = \sqrt{\langle e_{kl}\rangle\langle e_{kl}\rangle}$; the angle brackets denote averaging over the volumes of the constituents and over the entire volume of the composite.

The choice of reference-body parameters consists in determining the dependences of μ and K on the moduli μ_s and K_s of the constituents and concentrations c_s of the constituents. From dimensional

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considerations it follows that this dependence should be linear in the moduli of the constituents, and, in the general case, it can be written as

$$\mu = \sum_{s=1}^{n} \mu_s(e_s)p_s(c_s), \qquad K = \sum_{s=1}^{n} K_s p_s(c_s). \tag{2}$$

Here $p_s(c_s)$ are functions that determine the binding of the constituents. The three types of binding of the constituents considered in [4] are particular cases of the general formulas (2). Indeed, if $p_1 = 1$ and $p_s = 0$ ($s = 2, 3, \ldots, n$), then $\mu = \mu_1$ and $K = K_1$ and we obtain a model composite in which the first constituent acts as a binding matrix, and the remaining act as individual inclusions. If we set $p_s = c_s$ ($s = 1, 2, \ldots, n$), we have

$$\mu = \langle\mu\rangle, \quad K = \langle K\rangle$$

and system (2) describes the behavior of a multiphase mixture in which the constituents form mutually penetrating frameworks. Finally, if we set $p_s = c_s$ ($s = 1, 2, \ldots, m$) and $p_s = 0$ ($s = m + 1, m + 2, \ldots, n$), then $\mu = \sum_{s=1}^{m} c_s\mu_s$ and $K = \sum_{s=1}^{m} c_s K_s$ and we obtain a model composite in which the matrix is formed by the first m mutually penetrating constituents, and the remaining $(n - m)$ constituents are individual inclusions.

Thus, choice of the quantities μ and K reduces to determination of the functions $p_s(c_s)$, which are not independent and satisfy the relation

$$\sum_{s=1}^{n} p_s = 1, \tag{3}$$

which follows from formulas (2) if in the latter one sets $\mu_1 = \mu_2 = \ldots = \mu_n = \mu$ and $K_1 = K_2 = \ldots = K_n = K$. The values of p_s do not depend on the deformed states of the composite constituents and remain unchanged during loading from the initial to the last moment of deformation. Since for small elastoplastic strains, the structure of the composite material does not change, it is obvious that the values of p_s can be determined in the elastic deformation region and then used over the entire range of strain. Consequently, an adequate description of the elastoplastic behavior of composites requires additional information on their effective elastic properties. Therefore, in what follows, we assume that the experimental values of the effective elastic modulus of the composite $\mu_{\exp}$ and $K_{\exp}$ are known. Using the general Eqs. (1), we set up the desired function

$$W(p_s) = (\mu_e^* - \mu_{\exp})^2 + (K_e^* - K_{\exp})^2 \ [\mu_e^* = \mu_e(1-\alpha_e)\xi_e/(1-\alpha_e\xi_e], \quad K_e^* = K_e(1-\gamma_e)\eta_e/(1-\gamma_e\eta_e)). \tag{4}$$

Minimization of this function using condition (3) and relation (2) gives the values of the binding parameters p_s, which are then used for the calculation of the elastoplastic behavior of the composite. Here the subscript e indicates the values in the region of elastic strains.

We shall use the above procedure of estimating the binding of the constituents to refine the calculations of the elastoplastic deformation of a two-component composite containing individual inclusions. In this case, relations (2) take the form

$$\mu = \mu_1(1 - p) + \mu_2 p, \quad K = K_1(1 - p) + K_2 p, \quad p = p_2 \quad (0 \leqslant p \leqslant 1), \tag{5}$$

and the portion of the extension diagram of the matrix material is approximated by the exponential dependence

$$\sigma = s_1\left(1 - \exp\left(-\frac{E_1}{s_1}\varepsilon\right)\right), \tag{6}$$

where σ and ε are tensile stresses and strains, E_1 is the Young's modulus of the matrix material, and s_1 is the limit stress in the portion of the tensile diagram for the material matrix (the yield point).

The binding parameter p is obtained by numerical solution of system (4) and (5). Next, the elastoplastic behavior of the composite is calculated by formulas (1) with allowance for relations (6). In each step of macroscopic elastoplastic strains, the values of e_s are found numerically on a computer by the method of successive approximations.

The calculations were compared with the known model matrix–spherical inclusions ($p = 0$) and with experimental data on extension of specimens of epoxy resin filled with glass microspheres [5]. The behavior of the inclusions was considered ideally elastic. The glass volume content is $c_2 = 0.24$. The experimental value of the effective Young's modulus is $E^*_{\rm exp} = 6129$ MPa. The calculated values of the quantities are as follows: $E_1 = 3069$ MPa, $E_2 = 73{,}545$ MPa, $\nu_1 = 0.45$, $\nu_2 = 0.21$, $s_1 = 69$ MPa. The calculated value of the binding parameter is $p = 0.264$.

In Fig. 1, the results of calculations by formulas (1), (5), and (6) (curve 1) are compared with the experimental strain curve for epoxy resin filled with glass microspheres [5] (curve 2) and with the results of calculation by the model matrix — spherical inclusions ($p = 0$) (curve 3).

Thus, allowance for additional information on the effective elastic properties of a composite refines considerably the model of inelastic deformation and gives better agreement between experimental and theoretical data.

Disoriented reinforcement of composites by short fibers or particles leads to macroscopic anisotropy, and, hence, the proposed model is inapplicable in this case.

To describe adequately randomly reinforced composites by statistical averaging of the system of equilibrium equations, one should take into account the shape of inclusions and the statistical distribution of their orientations in the matrix. If the shapes of the fibers or particles are approximated by ellipsoids with a specified ratio of the semi-axes, the constitutive equations of macroscopic elastoplastic deformation for such a composite have the form [6]

$$\langle \sigma_{ij} \rangle = E^*_{ijkl}(e_{m,f}) \langle \varepsilon_{kl} \rangle, \tag{7}$$

where

$$E^*_{ijkl} = 2\mu_m I_{ijkl} + \delta_{ij}\delta_{kl}\lambda_m + c_f(2[\mu] I_{ijpq} + \delta_{ij}\delta_{pq}[\lambda]) a_{pqkl};$$

$$a_{ijkl} = (I_{ijpq} + R_{ijpq})^{-1} R_{pqkl}; \quad R_{ijkl} = (c_f c_m)^{-1} \sum_{s=1}^{n} c_s Q^{(s)}_{ijkl}; \quad Q^{(s)}_{ijkl} = (I_{ijkl} + P^{(s)}_{ijkl})^{-1};$$

$$P^{(s)}_{ijkl} = \frac{1}{2\mu_m} \left(2[\mu] Z^{(s)}_{ijkl} + \delta_{kl}[\lambda] Z^{(s)}_{ijpp}\right); \quad Z^{(s)}_{ijkl} = S^{(s)}_{ijkl} - \frac{\nu_m}{1+\nu_m} S^{(s)}_{ppkl}\delta_{ij};$$

$S^{(s)}_{ijkl}$ are the Eshelby-tensor components written in the laboratory coordinate system of ellipsoids of the sth direction, $[g] = g_f - g_m$, $\lambda = K - (2/3)\mu$, $c_{f,m}$ are the volume contents of the inclusions and the matrix, c_s is the volume content of ellipsoids of the sth direction, the subscript m refers to the matrix material and the subscript f to the inclusion material.

From relation (7) it follows that the effective tensor of the plasticity moduli E^*_{ijkl} generally depends on the plastic-shear moduli and the bulk moduli of the matrix and inclusion materials, on the fiber concentration, and also on the shape of the ellipsoidal inclusions, which is characterized by the ratio of the semi-axes of the ellipsoids a_1, a_2, and a_3:

$$E^*_{ijkl} = E^*_{ijkl}(\mu_{m,f}(e), K_{m,f}, c_f, \xi_1, \xi_2).$$

As a rule, the filler particles or fibers have a significant spread (up to 50%) of the relative sizes, which cannot be classified by orientations in designing a composite. Therefore, to obtain refined calculations by the model of elastoplastic deformation of a composite, it is reasonable to estimate the effective values of the parameters $\xi_{1,2} = a_{1,2}/a_3$ invoking additional information on the elastic properties of the composite.

We set up the desired function for the parameters ξ_1 and ξ_2:

$$W(\xi_1, \xi_2) = \sum_{i,j,k,l} (E^{e*}_{ijkl} - E^{\rm exp}_{ijkl})^2. \tag{8}$$

Here $E^{\rm exp}_{ijkl}$ are experimental values of the effective tensor of the elastic moduli.

Minimization of the desired function W gives a system of equations for determining the parameters ξ_1 and ξ_2, which can then be used in relations (7) to calculate elastoplastic properties.

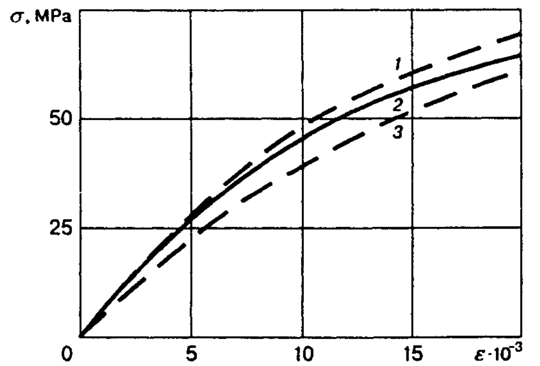

Fig. 1

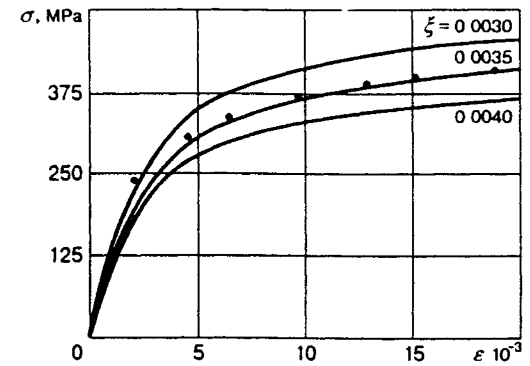

Fig. 2

To illustrate the proposed method of individual prediction, we consider the case of equiprobable orientation of ellipsoidal inclusions in the composite ($c_1 = c_2 = \ldots = c_n$). In this case, relations (7) take the form

$$\langle s_{ij}\rangle = 2\mu^*(e_{m,f})\langle e_{ij}\rangle, \qquad \langle \sigma_{ij}\rangle = 3K^*(e_{m,f})\langle \varepsilon_{ij}\rangle, \tag{9}$$

where $\mu^* = \mu_m + [\mu]c_f\alpha/(c_m + c_f\alpha)$, $K^* = K_m + [K]c_f\gamma/(c_m + c_f\gamma)$, $\gamma = \alpha - 3\beta$, and $\alpha = (1/15)(3Q_{pqpq} - Q_{ppqq})$ and $\beta = (1/15)(Q_{pqpq} - 2Q_{ppqq})$ are invariants of the tensor Q_{ijkl}.

Equations (9) were used to calculate the elastoplastic properties of composite specimens fabricated from a sintered aluminum powder (SAP) — an aluminum matrix with randomly distributed particles of aluminum oxide Al_2O_3 (14%) formed by sintering of the aluminum powder. These particles are plates with thickness $h = 0.055$ μm and linear dimension $0 \leqslant L \leqslant 16$ μm in plan [7]. The oxide particles are approximated by ellipsoids of revolution (flattened ellipsoids) for which the ratio of the semi-axes is $\xi = hL^{-1}$. In this case, the Eshelby tensor components are expressed in terms of elementary functions, and the calculation of the invariants α and γ is readily performed on a computer [8].

The tensile diagram of the composite considered is plotted by the formula

$$\langle \sigma_{11}\rangle = \frac{9K^*\mu^*}{3K^* + \mu^*}\langle \varepsilon_{11}\rangle. \tag{10}$$

The portion of the tensile diagram of the aluminum matrix is approximated by the exponential dependence of [4], for which the function $\mu_m(e)$ has the form

$$\mu_m(e) = \frac{k_m}{2e}\left(1 - \exp\left(-\frac{2G_m e}{k_m}\right)\right). \tag{11}$$

Here G_m is the shear modulus and k_m is the limit shear stress in the given region (the yield point). The inclusion material (high-strength and high-modulus aluminum oxide particles) is considered ideally elastic over the entire deformation process: $\mu_f = \text{const}$.

The calculated values are as follows: $E_m = 71$ MPa, $E_f = 2500$ GPa, $\nu_m = 0.34$, $\nu_f = 0.2$, $k_m = 25$ MPa and $c_f = 0.14$. The experimental value of the effective Young's modulus is $E_{\text{exp}} = 1750$ GPa. The calculated value of the parameter $\xi = 0.0035$. Equations (9)–(11) were solved numerically on a computer by the method of successive approximations.

Figure 2 shows a comparison of the theoretical tensile diagrams of SAP calculated by formulas (9)–(11) with the experimental results of [9] [the curves correspond to the calculations by formulas (9)–(11), and the points correspond to the experimental data].

Thus, allowance for additional information on the elastic properties of a composite allows one to estimate with sufficient accuracy the subsequent elastoplastic behavior of the composite.

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